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JUSTIFICATION OF REQUIREMENTS FOR A STRUCTURAL EFFICIENCY INDICATOR OF UNEQUAL-ENERGY COMPLEX SIGNAL ENSEMBLES IN CODE DIVISION MULTIPLE ACCESS SYSTEMS

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ОБҐРУНТУВАННЯ ВИМОГ ЩОДО ПОКАЗНИКА СТРУКТУРНОЇ ЕФЕКТИВНОСТІ АНСАМБЛІВ РІЗНОЕНЕРГЕТИЧНИХ СКЛАДНИХ СИГНАЛІВ У СИСТЕМАХ МНОЖИННОГО ДОСТУПУ З КОДОВИМ РОЗДІЛЕННЯМ

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Abstract. *The characteristics of a code division multiple access system significantly depend on the choice of an ensemble of complex signals. The structure of the complex signals in the ensemble is taken into account through the cross-correlation of the signals and the signal energies in the ratio of useful signal energy to the total energy of multiple-access interference and noise. For ensembles of equal-energy complex signals, the available efficiency indicators make it possible to estimate the influence of the ensemble on multiple-access interference without separately taking into account the signal energy distribution, since the signals can be normalized to unit energy. For ensembles of unequal-energy complex signals, the signal energy distribution affects the level of multiple-access interference together with the cross-correlation of the signals. The purpose of the article is to substantiate the requirements for the structural efficiency indicator of ensembles of unequal-energy complex signals in code division multiple access systems on the basis of analysis of multiple-access interference energy and the ratio of useful signal energy to the total energy of multiple-access interference and noise. It is established that, for assessing complex signal ensembles, it is appropriate to distinguish between indicators through which the ratio of useful signal energy to the total energy of multiple-access interference and noise can be expressed directly and general squared-correlation functionals that do not provide such a direct expression. For equal-energy ensembles, the assessment of structural efficiency can be reduced to the cross-correlations of the signals, and the structural efficiency indicator can be the product of the signal ensemble size and the square of the maximum ensemble-wide value of cross-correlation. For unequal-energy ensembles, the total energy of multiple-access interference is not reduced to a form analogous to the equal-energy case, because it is simultaneously affected by the cross-correlations of the signals and by the nonuniform distribution of their energies. Taking into account the analysis performed, the following requirements should be imposed on the structural efficiency indicator of unequal-energy complex signal ensembles: the indicator should be related to the signal-to-interference-plus-noise ratio, be sensitive to unfavorable cases of multiple-access interference realization, and be suitable for ranking and comparing unequal-energy complex signal ensembles of different sizes, while remaining consistent with the equal-energy limiting case.*

Keywords: *structural efficiency indicator, unequal-energy complex signal ensemble, code division multiple access system, multiple-access interference, cross-correlation, total squared correlation.*

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Анотація. *Характеристики системи множинного доступу з кодовим розділенням істотно залежать від вибору ансамблю складних сигналів. Структура складних сигналів ансамблю врахована через взаємну кореляцію сигналів та енергії сигналів у відношенні енергії корисного сигналу до сумарної енергії завад множинного доступу і шуму. Для ансамблів рівноенергетичних складних сигналів наявні показники ефективності дають змогу оцінювати вплив ансамблю на завади множинного доступу без окремого врахування розподілу енергії сигналів через можливість їх нормування до одиничної енергії. Для ансамблів різноенергетичних складних сигналів розподіл енергії сигналів впливає на рівень завад множинного доступу разом із взаємною кореляцією сигналів. Метою статті є обґрунтування вимог щодо показника структурної ефективності ансамблів різноенергетичних складних сигналів систем множинного доступу з кодовим розділенням на основі аналізу енергії завад множинного доступу та відношення енергії корисного сигналу до сумарної енергії завад множинного доступу і шуму. Установлено, що, оцінюючи ансамблі складних сигналів, доцільно розрізняти показники, через які відношення енергії корисного сигналу до сумарної енергії завад множинного доступу і шуму може бути виражене безпосередньо, і кореляційні функціонали загальної квадратичної кореляції, які такого безпосереднього вираження не забезпечують. Для рівноенергетичних ансамблів оцінювання структурної ефективності може бути зведено до взаємних кореляцій сигналів, а показником структурної ефективності може бути добуток об'єму ансамблю сигналів на квадрат максимального за ансамблем значення взаємної кореляції. Для різноенергетичних ансамблів сумарна енергія завад множинного доступу не зведена до форми, аналогічної рівноенергетичному випадку, оскільки на неї одночасно впливають взаємні кореляції сигналів і нерівномірність розподілу їхніх енергій. З урахуванням проведеного аналізу до показника структурної ефективності ансамблів різноенергетичних складних сигналів доцільно висунути такі вимоги: показник повинен мати зв’язок із відношенням енергії корисного сигналу до сумарної енергії завад множинного доступу і шуму, бути чутливим до несприятливих випадків реалізації завад множинного доступу і бути придатним для ранжування і порівняння різноенергетичних ансамблів складних сигналів різного об’єму зі збереженням коректного переходу до рівноенергетичного граничного випадку.*

Ключові слова: *показник структурної ефективності, ансамбль різноенергетичних складних сигналів, система множинного доступу з кодовим розділенням, завади множинного доступу, взаємна кореляція, загальна квадратична кореляція.*

Relevance of the research topic. A promising direction in the development of code division multiple access systems is the transition to high-frequency ranges, the introduction of ultra-wideband access modes, and the strengthening of requirements for energy efficiency and electromagnetic compatibility. The implementation of energy-efficient code division multiple access systems in high-frequency ranges can be provided by using ensembles of impulse complex signals, which make it possible to build high-frequency signal receiving, transmitting, and processing paths according to simplified schemes suitable for mass production. The impulse nature of such signals and the simplification of high-frequency paths form the technical basis for the energy efficiency of these systems. In [1], ensembles of periodic impulse sequences are proposed that provide a low level of multiple-access interference under arbitrary time shifts between signals, which is an important factor for the practical implementation of asynchronous operation of a multiple access system. For impulse complex signal ensembles, unequal signal energy may arise from different numbers of impulses in individual signals [1]. The available evaluation indicators are mainly associated with the equal-energy case.

Therefore, substantiating the requirements for a structural efficiency indicator of constructively unequal-energy impulse complex signal ensembles, which reflects their structural-energy properties and is suitable for selecting, comparing, and ranking constructively unequal-energy complex signal ensembles, is a relevant task.

Introduction. The characteristics of a code division multiple access system significantly depend on the selected complex signal ensemble. The ensemble determines the structure of complex signals, the maximum number of users through the ensemble size, and the maximum level of multiple-access interference to one user from all signals of other users. Therefore, before introducing a structural efficiency indicator, it is necessary to relate the structural properties of the ensemble to the energy of multiple-access interference.

For equal-energy complex signal ensembles, available efficiency indicators make it possible to assess the influence of the ensemble on multiple-access interference without separately taking into account the distribution of signal energies, due to the possibility of normalizing them to unit energy. For unequal-energy complex signal ensembles, the distribution of signal energies affects the level of multiple-access interference together with the cross-correlation of signals. Therefore, before defining a structural efficiency indicator for such signal ensembles, it is necessary to substantiate the requirements for an indicator that must take into account the structural-energy properties of the ensemble.

Analysis of recent research and publications. The characteristics of a code division multiple access system significantly depend on the selected complex signal ensemble, which determines the structure of complex signals as a set of signal components, their parameters, their placement on the time axis within the signal duration, the maximum number of users through the ensemble size, and the maximum level of multiple-access interference to one user from all other user signals, that is, intrasystem interference. The structure of the complex signals in the ensemble directly determines the level of multiple-access interference and is taken into account through signal correlation and signal energies in the ratio of useful signal energy to the total energy of multiple-access interference and noise (SINR) [2–5, 8, 9].

Since noise is not taken into account when synthesizing complex signal ensembles, ranking them, and selecting them, the task arises of determining the influence of signal ensembles on the characteristics of a code division multiple access system, which in this paper will be referred to as the efficiency assessment of complex signal ensembles.

Without noise, the ratio of useful signal energy to interference energy can be expressed through cross-correlations of signals or through the maximum cross-correlation value over the ensemble [2–5, 7–12]. Ensemble-wide squared-correlation functionals do not directly express the ratio for an individual user in the general case [12, 14, 17].

For equal-energy signals, signal energies can be normalized or treated as a common factor; therefore, correlation quantities become the main variables of comparison [2–5, 7–13, 15, 16]. For such signals, their energies are not independent distinguishing quantities due to their equality and the possibility of normalization by a common value. In this case, comparison can be reduced to cross-correlations of signal pairs or to their maximum value over the ensemble [2–5, 7, 11, 12]. The Welch bound gives a theoretical lower level of possible cross-correlation for a given ensemble size and signal-space dimension, but it does not select a concrete structurally specified ensemble [7, 11–13, 15, 16].

In contrast to the equal-energy case, the energies of an unequal-energy signal ensemble are significant quantities in the assessment of ensemble efficiency, because the components of multiple-access interference energy are products of signal energies and squared cross-correlations. Because signal energies are unequal, this sum cannot be reduced to one common energy factor. Unequal user energies are usually associated with propagation conditions, near-far effect, power control, or user power constraints [3–6, 18–20]. Structurally determined energy distribution is a different case. Thus, unequal energy caused by the signal structure requires a separate formulation within the present assessment task.

Squared-correlation functionals characterize the ensemble as a whole [12, 14, 17]. These functionals depend on the full set of cross-correlations in the ensemble. They cannot be replaced by only the maximum correlation value without losing the ensemble-wide meaning. For equal-energy ensembles, total squared correlation (TSC) is a theoretical correlation criterion connected with the Welch bound [7, 12–14]. For weighted formulations, total weighted squared correlation can be used as an auxiliary ensemble-wide comparison quantity [14, 17].

The analysis covers equal-energy signal sets, correlation bounds, total weighted squared correlation, and unequal-power cases. However, for structurally unequal-energy complex signal ensembles, a separate formulation of requirements remains necessary.

Defining the purpose and objectives of the research. The purpose of the article is to substantiate the requirements for a structural efficiency indicator of unequal-energy complex signal ensembles in code division multiple access systems on the basis of analysis of multiple-access interference energy and the ratio of useful signal energy to the total energy of multiple-access interference and noise. The objectives corresponding to this purpose are to analyze the expression for multiple-access interference energy for equal-energy and unequal-energy signal ensembles, to clarify the role of squared-correlation functionals, and to formulate requirements for the structural efficiency indicator of unequal-energy complex signal ensembles.

The main part of the research. For further analysis, we will use the general model of correlation reception in a code division multiple access system [2–5, 8, 9]. For the signal of the i-th user, we define the sample at the receiver output in the energy form $E_{Z,i}$:

$$E_{Z,i} = E_i + \sum_{j=1, j\neq i}^{L} E_j \left(R_{ij}^{(max)}\right)^2 + E_{N,i},$$

where E_i – energy of the useful signal of the i-th user, $i = 1, \dots, L$;

E_j – energy of the j-th signal of the ensemble, $j = 1, \dots, L, j \neq i$;

$R_{ij}^{(max)}$ – cross-correlation value of signals i and j for the ensemble, $R_{ij}^{(max)} = \max_{\tau} R_{ij}[\tau]$, $R_{ij}[\tau]$ – cross-correlation value of signals i and j at mutual time shift τ;

$E_{N,i}$ – noise energy for the i-th user;

L – size of the signal ensemble, which determines the maximum number of users of the system.

The cross-correlation $R_{ij}[\tau]$ for the pair of signals i and $j, j \neq i$, at the mutual time shift τ is defined as [2–5, 10–12]

$$R_{ij}[\tau] = \frac{\sum_k s_i[k] s_j[k-\tau]}{\sqrt{E_i E_j}},$$

where $s_i[k]$ – sample of signal i at time instant k;

$s_j[k-\tau]$ – sample of signal j shifted by τ relative to time instant k;

τ – mutual time shift between signals i and $j, j \neq i$.

It should be emphasized that the set of cross-correlation values of signals i and j for the ensemble as a whole is determined by the maximum value of $R_{ij}[\tau]$ for this pair of signals over the mutual time shift:

$$R_{ij}^{(\max)} = \max_{\tau} R_{ij}[\tau].$$

For $i = j$, the quantity $R_{ij}[\tau]$ defines the autocorrelation of the i-th signal. For zero mutual time shift between the i-th signal and its reference copy in the receiver, we have $R_{ii}[0]=1$. Cases for which $R_{ii}\neq 1$ are associated with nonideal signal synchronization in the receiver and are not considered in this paper.

In the expression for $E_{Z,i}$, the second term is the total energy of multiple-access interference, or intrasystem interference, MAI (Multiple Access Interference) for the i-th user $E_{I,i}$ from the signals of the j-th users, $j\neq i$:

$$E_{I,i}=\sum_{j=1,j\neq i}^{L}E_j\left(R_{ij}^{(max)}\right)^2.$$

Each term of the expression for $E_{I,i}$ is determined by the product of the energy of the j-th interfering signal and the square of the cross-correlation value of the i-th and j-th signals, $E_j\left(R_{ij}^{(max)}\right)^2$. If for the unequal-energy signal ensemble as a whole we set

$$R_{max}^2=\max_{i,j,i\neq j}\left(R_{ij}^{(max)}\right)^2,$$

then for each term in the expression for $E_{I,i}$, which corresponds to each pair of signals and contains the energy multiplier E_j, the following inequality holds:

$$E_j\left(R_{ij}^{(max)}\right)^2\leq E_jR_{max}^2.$$

As a result, for the total multiple-access interference energy $E_{I,i}$, we obtain the upper estimate $E_{I,i}^{(max)}$:

$$E_{I,i}^{(max)}=\sum_{j=1,j\neq i}^{L}E_j\left(R_{ij}^{(max)}\right)^2\leq R_{max}^2\sum_{j=1,j\neq i}^{L}E_j.$$

After defining the total energy of all signals of the ensemble as $E_\Sigma=\sum_{k=1}^{L}E_k$, we obtain

$$E_{I,i}^{(max)}=R_{max}^2(E_\Sigma-E_i).$$

The obtained relation for $E_{I,i}^{(max)}$ specifies a value that the multiple-access interference energy $E_{I,i}$ cannot exceed. Thus, for unequal-energy signal ensembles, the upper estimate of the multiple-access interference energy $E_{I,i}^{(max)}$ depends on the largest squared cross-correlation over the ensemble R_{max}^2 and on the total energy of all interfering signals $(E_\Sigma-E_i)$.

For an equal-energy signal ensemble $E_j=E, j=1,\dots,L$, the multiple-access interference energy MAI for the i-th user is determined by the product of the signal energy and the sum of squared cross-correlations of the i-th signal with all other ensemble signals:

$$E_{I,i}=E\sum_{j=1,j\neq i}^{L}\left(R_{ij}^{(max)}\right)^2.$$

In this case, the multiple-access interference energy no longer depends on the individual energy values of the signals of separate users, because these energies are identical for all signals of the ensemble. Therefore, differences in the values of $E_{I,i}$ for different users are determined not by separate signal energies but by the sum of cross-correlation values of the signal of the i-th user with the signals of all other users.

If for the equal-energy signal ensemble as a whole we set

$$R_{max}^2=\max_{i,j,i\neq j}\left(R_{ij}^{(max)}\right)^2,$$

then for each term in the expression for $E_{I,i}$, which corresponds to each pair of signals and contains the same multiplier E, the following inequality holds:

$$E\left(R_{ij}^{(max)}\right)^2 \leq ER_{max}^2.$$

As a result, we have the upper estimate $E_{I,i}^{(max)}$ for the total multiple-access interference energy:

$$E_{I,i}^{(max)} = E\sum_{j=1,j\neq i}^{L}\left(R_{ij}^{(max)}\right)^2 \leq E\sum_{j=1,j\neq i}^{L}R_{max}^2,$$

$$E_{I,i}^{(max)} = (L-1)ER_{max}^2,$$

which is determined by the signal energy of the ensemble E, the number of interfering signals $L-1$, and the largest squared cross-correlation over the ensemble R_{max}^2.

The obtained estimates $E_{I,i}^{(max)}$ can be used for assessing the efficiency of signal ensembles, their ranking, and their comparison. For equal-energy signal ensembles with the same values of E and L, the comparison is simplified and can be reduced to the value R_{max}^2, because the energy multiplier is common for all ensemble signals. For unequal-energy signal ensembles, the estimate $E_{I,i}^{(max)}$ can also be used for ranking and comparison; however, together with the quantity R_{max}^2, the distribution of energies among the ensemble signals must also be taken into account.

It should be noted that, unlike an equal-energy signal ensemble, where the common energy multiplier is factored outside the summation sign and the reduction is performed to the sum of squared cross-correlations, in the unequal-energy case each term contains its own energy multiplier. Therefore, a common multiplier cannot be separated in the sum, and the expression for $E_{I,i}$ for an unequal-energy signal ensemble is not obtained by direct analogy with the equal-energy case.

Next, we give the expression for the ratio of the useful signal energy of the i-th user to the total energy of multiple-access interference and noise, $SINR_i$ [2–5, 8, 9]:

$$\mathrm{SINR_i} = \frac{E_i}{E_{I,i}+E_{N,i}}.$$

After substituting the general relation for $E_{I,i}$, we obtain

$$\mathrm{SINR_i} = \frac{E_i}{\sum_{j=1,j\neq i}^{L}E_j\left(R_{ij}^{(max)}\right)^2+E_{N,i}} = \frac{1}{\sum_{j=1,j\neq i}^{L}\frac{E_j}{E_i}\left(R_{ij}^{(max)}\right)^2+\frac{E_{N,i}}{E_i}}.$$

We consider the effect of signal dominance in an unequal-energy ensemble. From the expression for $\mathrm{SINR_i}$, it follows that the contributions of interfering signals to the denominator are determined by the ratios of their energies to the energy of the useful signal E_j/E_i, weighted by squared cross-correlations. At equal or close values of cross-correlations, the multiple-access interference energy increases as the energy of the interfering signal increases. Therefore, the most critical influence of the dominance effect appears for users with lower useful-signal energies, for which the decrease in $\mathrm{SINR_i}$ will be larger than for users with higher useful-signal energies.

To separate the influence of noise, we set $E_{N,i} = 0$. Then SINR$_\mathrm{i}$ will be determined only by the relative energy contributions of the interfering signals and their correlation multipliers:

$$\mathrm{SINR}_\mathrm{i}^{(\mathrm{E_{N,i}=0})} = \frac{E_i}{\sum_{j=1, j\neq i}^{L} E_j \left(R_{ij}^{(max)}\right)^2} = \frac{1}{\sum_{j=1, j\neq i}^{L} \frac{E_j}{E_i} \left(R_{ij}^{(max)}\right)^2}.$$

For an equal-energy signal ensemble, when $E_j = E_i = E, j = 1, \ldots, L, j \neq i,$ all relative energy contributions in the denominator take the unit value. Then $\mathrm{SINR}_\mathrm{i}^{(\mathrm{E_{N,i}=0})}$ depends only on the sum of squared cross-correlations of the i-th signal with all other signals of the ensemble:

$$\mathrm{SINR}_\mathrm{i}^{(\mathrm{E_{N,i}=0})} = \frac{1}{\sum_{j=1, j\neq i}^{L} \left(R_{ij}^{(max)}\right)^2}.$$

To pass to the worst-case estimate in an equal-energy ensemble, we use the fact that in the sum the square of the maximum cross-correlation value for each term does not exceed R_{max}^2:

$$\sum_{j=1, j\neq i}^{L} \left(R_{ij}^{(max)}\right)^2 \leq (L-1) R_{max}^2.$$

In this case, the largest denominator in the expression for $\mathrm{SINR}_\mathrm{i}^{(\mathrm{E_{N,i}=0})}$ corresponds to its smallest value. Therefore, the upper estimate for the sum of squared cross-correlations is reduced to a scalar estimate of the worst case SINR$_\mathrm{min}$:

$$\mathrm{SINR}_\mathrm{min}^{(\mathrm{E_{N,i}=0})} = \frac{1}{(L-1) R_{max}^2}.$$

Thus, the quantity $\mathrm{SINR}_\mathrm{min}^{(\mathrm{E_{N,i}=0})}$ determines SINR at $E_{N,i} = 0$ for the worst case for an equal-energy ensemble through the number of interfering signals and the square of the largest cross-correlation value.

For $L \gg 1$, the difference between $L - 1$ and L is relatively small, and therefore the further transition is performed through the approximation $(L-1) R_{max}^2 \approx L R_{max}^2$:

$$\mathrm{SINR}_\mathrm{min}^{(\mathrm{E_{N,i}=0})} \approx \frac{1}{L R_{max}^2}.$$

This makes it possible to obtain a simplified convenient form for determining $\mathrm{SINR}_\mathrm{min}^{(\mathrm{E_{N,i}=0})}$. It should be noted that $\mathrm{SINR}_\mathrm{min}^{(\mathrm{E_{N,i}=0})}$ is a scalar and is determined by the product of such structural characteristics of the signals as the ensemble size L and the maximum squared cross-correlation value over the ensemble R_{max}^2. Therefore, the product $L R_{max}^2$ can be used as a structural efficiency indicator of an equal-energy ensemble, because it simultaneously takes into account the number of signals in the ensemble and the worst correlation case. Unlike indicators determined only by cross-correlations of signals, the quantity $L R_{max}^2$ remains sensitive to changes in the ensemble size. Therefore, it can be used to compare ensembles of different sizes according to the worst case of multiple-access interference realization. Thus, we introduce the structural efficiency indicator of an equal-energy ensemble γ_str as

$$\gamma_\mathrm{str} = L R_{max}^2.$$

Then the expression for $\mathrm{SINR}_\mathrm{min}^{(\mathrm{E_{N,i}=0})}$ takes the form

$$\mathrm{SINR}_{\mathrm{min}}^{(\mathrm{E}_{\mathrm{N,i}}=0)} \approx \frac{1}{\gamma_{\mathrm{str}}}.$$

Thus, the indicator γ_{str} is inversely related to the worst-case SINR value in the absence of noise. As the value of γ_{str} increases, the value of $\mathrm{SINR}_{\mathrm{min}}^{(\mathrm{E}_{\mathrm{N,i}}=0)}$ decreases; as it decreases, the value of $\mathrm{SINR}_{\mathrm{min}}^{(\mathrm{E}_{\mathrm{N,i}}=0)}$ increases. Therefore, γ_{str} can be used for ranking and comparing equal-energy ensembles according to the worst-case interference effect.

Next, we consider another ensemble-wide correlation characteristic: the TSC functional [12, 14–17]. Unlike γ_{str}, which is directly related to the worst value of $\mathrm{SINR}_{\mathrm{min}}^{(\mathrm{E}_{\mathrm{N,i}}=0)}$, the TSC functional for equal-energy signals takes into account both cross-correlations and autocorrelations of the signal ensemble:

$$TSC = \sum_{i=1}^{L}\sum_{j=1}^{L}\left(R_{ij}^{(max)}\right)^2.$$

For normalized autocorrelation terms, we have $\left(R_{ii}^{(max)}\right)^2 = 1$, therefore

$$TSC = L + \sum_{i=1}^{L}\sum_{j=1, j\neq i}^{L}\left(R_{ij}^{(max)}\right)^2.$$

In this form, the constant part of the functional is formed by the autocorrelation terms, and the variable part is formed by the set of squared cross-correlations. Therefore, for a fixed ensemble size L, the change in TSC is determined by the change in the value of the sum of cross-correlations.

At the same time, $\mathrm{SINR}_{\mathrm{i}}$ in the general case cannot be expressed directly through the TSC functional, because $\mathrm{SINR}_{\mathrm{i}}$ is determined by correlation contributions for the i-th signal, whereas the TSC functional accumulates such contributions over the whole ensemble. Therefore, the TSC functional should be treated as an auxiliary ensemble-wide correlation characteristic rather than as a direct worst-case interference indicator.

Thus, the indicator γ_{str} and the TSC functional characterize different properties of the signal ensemble. The indicator γ_{str} sets the scalar of the worst case and is directly related to the worst value of SINR; therefore, it can be used as a structural efficiency indicator of an equal-energy ensemble.

The TSC functional characterizes the overall correlation property of the ensemble as a whole. Therefore, for equal-energy ensembles it can be used as a theoretical correlation criterion, and for unequal-energy ensembles it can be used as an auxiliary comparison indicator in weighted forms. At the same time, the TSC functional cannot directly replace the indicator γ_{str}, because it takes into account pairwise contributions for the entire ensemble rather than for one maximally unfavorable interaction mode. Thus, the main part establishes the analytical basis for the requirements: the structural indicator should be related to the worst-case interference energy, while TSC remains an auxiliary ensemble-wide correlation characteristic.

Conclusions. The analysis performed shows that quantities directly related to the useful-signal-to-interference-and-noise ratio should be used as the basis for assessing the worst interference case, while total squared correlation and its weighted forms should be used as auxiliary ensemble-wide correlation characteristics.

For equal-energy complex signal ensembles, structural efficiency assessment can be reduced to cross-correlations of ensemble signals. For this case, the product of the ensemble size and the square of the maximum cross-correlation value over the ensemble is proposed as the structural efficiency indicator. This indicator determines the worst case for SINR without taking noise into account and can be used for ranking and comparing ensembles of different sizes.

For unequal-energy complex signal ensembles, the total multiple-access interference energy cannot be reduced to a form analogous to the equal-energy case. Because of this, the value of the multiple-access interference energy is simultaneously affected by cross-correlations of signals and by the nonuniform distribution of their energies. As a result, a structural efficiency indicator for unequal-energy ensembles cannot be obtained by direct transfer of the equal-energy form.

Taking into account the analysis performed, the following requirements should be imposed on the structural efficiency indicator of unequal-energy complex signal ensembles: the indicator should be related to SINR, be sensitive to unfavorable cases of multiple-access interference realization, and be suitable for ranking and comparing unequal-energy complex signal ensembles of different sizes, while remaining consistent with the equal-energy limiting case.

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